A NEW RLS ALGORITHM BASED ON THE VARIATION CHARACTERISTICS OF A ROOM IMPULSE RESPONSE

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ABSTRACT

This paper proposes a new adaptive algorithm (called the ES-RLS algorithm) with double the convergence speed of the conventional RLS algorithm. Our previous report showed that the variation of a room impulse response becomes progressively smaller along the series by the same exponential ratio as the impulse response. The ES-RLS algorithm is derived by incorporating these variation characteristics into the conventional RLS algorithm using Kalman filter theory, which gives physical meaning to the RLS algorithm. The ES-RLS algorithm adjusts coefficients with large errors in large steps and coefficients with small errors in small steps. Computer simulations demonstrated that our new adaptive algorithm converged twice as fast as the conventional RLS algorithm.

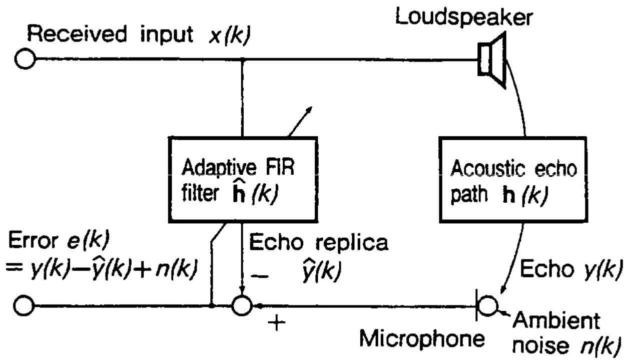

Fig. 1 Configuration of an acoustic echo canceller.

1. INTRODUCTION

Acoustic echo cancellers are now in use in teleconferencing and hands-free telecommunication systems to overcome acoustic feedback and to make conversation comfortable. An acoustic echo canceller adaptively identifies the impulse response between a loudspeaker and a microphone, and then produces an echo replica which is subtracted from the real echo (acoustic feedback signal). Since the impulse response varies when a person moves and varies with the environment, an adaptive filter is used for the identification.

The LMS algorithm and NLMS (normalized LMS) algorithm [1] require few computations, and are, therefore, widely applied for acoustic echo cancellers. However, there is a strong need to improve the convergence speed of the LMS and NLMS algorithms.

The RLS (recursive least-squares) algorithm [2], whose convergence does not depend on the input signal, is the fastest of all conventional adaptive algorithms. The major drawback of the RLS algorithm is its large computational cost. However, fast (small computational cost) RLS algorithms have been studied recently [3], and the RLS algorithm is expected to be used in acoustic echo cancellers in the near future.

We have studied the variation characteristics of a room impulse response, which is the " unknown system " for the acoustic echo canceller. We have reported that the expected variation of a room impulse response becomes progressively smaller along the series by the same exponential ratio as the impulse response energy decay [4]. As a result, we have proposed two adaptive algorithms. One is the ES (exponentially weighted stepsize NLMS) algorithm [4] which reflects the variation characteristics of a room impulse response in the conventional NLMS algorithm [1]. The other is the ESP (exponentially weighted stepsize projection) algorithm [5] which reflects the variation characteristics of a room impulse response in the conventional projection algorithm [6].

The basic concept of both the ES and ESP algorithms is to adjust coefficients with large errors in large steps and coefficients with small errors in small steps. For this purpose, these algorithms use a different stepsize for each coefficient of an adaptive transversal FIR filter by introducing a stepsize matrix with a diagonal form instead of the scalar stepsize of the conventional adaptive algorithms. These stepsizes are time-invariant and weighted proportional to the expected variation of the room impulse response. Consequently, the ES and ESP algorithms converge twice as fast as the conventional NLMS and projection algorithms, respectively.

In this paper we aim to obtain a faster algorithm by incorporating knowledge of the room impulse response into the RLS algorithm. Unlike the NLMS and projection algorithms, the RLS algorithm does not have a scalar stepsize. Therefore, the variation characteristics of a room impulse response cannot be reflected directly in the RLS algorithm. Here, we study the RLS algorithm from the viewpoint of the Kalman filter [2] because (a) the RLS algorithm can be regarded as a special version of the Kalman filter and (b) each parameter of the Kalman filter has a physical meaning. Then we propose the ES-RLS (exponentially weighted stepsize RLS) algorithm which reflects the variation characteristics of a room impulse response in the RLS algorithm. Computer simulations demonstrate that this algorithm converges twice as fast as the conventional RLS algorithm.

2. ACOUSTIC ECHO CANCELLER AND ADAPTIVE ALGORITHM

The configuration of an acoustic echo canceller is shown in Fig. 1. The echo canceller identifies the impulse response $\mathbf{h}(k)$ between the loudspeaker and the microphone at discrete time k. Usually, $\hat{\mathbf{h}}(k)$ is a transversal FIR filter. The FIR filter coefficient $\hat{\mathbf{h}}(k)$ should be a copy of the impulse response $\mathbf{h}(k)$. An echo replica $\hat{y}(k)$ is created by convolving $\hat{\mathbf{h}}(k)$ with the received input vector $\mathbf{x}(k)$, where $\mathbf{x}(k) = [x(k), x(k-1), \cdots, x(k-L+1)]^T$ and L represents the number of taps, and the superscript ^{T} represents the transpose of a vector (matrix). The echo replica $\hat{y}(k)$ is then subtracted from the real echo $y(k)$ to give the error $e(k) = y(k) - \hat{y}(k) + n(k)$, where $n(k)$ represents the ambient noise. The adaptive FIR filter $\hat{\mathbf{h}}(k)$ is adjusted to decrease the error power in every sampling interval. The adaptive algorithm should provide fast convergence and high echo return loss enhancement (*ERLE*, defined as the ratio of the real echo power to the error power excluding the ambient noise).

3. KALMAN FILTER AND RLS ALGORITHM

3.1 Kalman filter

The Kalman filter, when used as an adaptive filter, updates the filter coefficient vector $\hat{\mathbf{h}}(k)$ according to the following equations [2].

$$\hat{\mathbf{h}}(k+1) = \hat{\mathbf{h}}(k) + \mathbf{k}(k)e(k) \tag{1}$$

$$\mathbf{k}(k) = \frac{\mathbf{P}_K(k)\mathbf{x}(k)}{R(k) + \mathbf{x}(k)^T\mathbf{P}_K(k)\mathbf{x}(k)} \tag{2}$$

$$\tilde{\mathbf{P}}_K(k) = \mathbf{P}_K(k) - \mathbf{k}(k)\mathbf{x}(k)^T\mathbf{P}_K(k) \tag{3}$$

$$\mathbf{P}_K(k+1) = \tilde{\mathbf{P}}_K(k) + \mathbf{Q}(k) \tag{4}$$

$$e(k) = y(k) - \hat{\mathbf{h}}(k)^T\mathbf{x}(k) + n(k), \tag{5}$$

where

- $\mathbf{k}(k)$: *L-th* order Kalman gain vector,
- $\mathbf{P}_K(k) = E[\{\mathbf{h}(k) - \hat{\mathbf{h}}(k)\}\{\mathbf{h}(k) - \hat{\mathbf{h}}(k)\}^T]$: $L \times L$ a priori coefficient error covariance matrix,
- $\tilde{\mathbf{P}}_K(k) = E[\{\mathbf{h}(k) - \hat{\mathbf{h}}(k+1)\}\{\mathbf{h}(k) - \hat{\mathbf{h}}(k+1)\}^T]$: $L \times L$ a posteriori coefficient error covariance matrix,
- $\mathbf{Q}(k) = E[\Delta\mathbf{h}(k)\Delta\mathbf{h}(k)^T]$: covariance matrix of the impulse response variation $\Delta\mathbf{h}(k)$ at time step k, $\mathbf{h}(k+1) = \mathbf{h}(k) + \Delta\mathbf{h}(k)$,
- $R(k) = E[n(k)^2]$: power of the ambient noise $n(k)$,
- $E[\cdot]$: statistical expectation.

Equation (3) represents the relationship between $\mathbf{P}_K(k)$ and $\tilde{\mathbf{P}}_K(k)$. It shows that the coefficient error covariance matrix $\mathbf{P}_K(k)$ decreases according to the update of (1) and becomes $\tilde{\mathbf{P}}_K(k)$. On the other hand, equation (4) shows that according to the impulse response variation $\mathbf{h}(k+1) = \mathbf{h}(k) + \Delta\mathbf{h}(k)$, the covariance matrix $\mathbf{Q}(k)$ of the impulse response variation $\Delta\mathbf{h}(k)$ is added to $\tilde{\mathbf{P}}_K(k)$ and increases to become $\mathbf{P}_K(k+1)$.

In the Kalman filter, the Kalman gain $\mathbf{k}(k)$ is calculated according to (2) using the coefficient error covariance matrix $\mathbf{P}_K(k)$ calculated one time step before. The filter coefficient vector $\hat{\mathbf{h}}(k)$ is adjusted according to (1). According to the adjustment, the filter coefficient $\hat{\mathbf{h}}(k+1)$ is obtained that minimizes the sum of the mean-squared coefficient error $E[\{\mathbf{h}(k+1) - \hat{\mathbf{h}}(k+1)\}^T\{\mathbf{h}(k+1) - \hat{\mathbf{h}}(k+1)\}]$ at time step $k+1$.

As described above, in the adaptive processing by the Kalman filter, the covariance matrix $\mathbf{Q}(k)$ of the impulse response variation $\Delta\mathbf{h}(k)$ and power $R(k)$ of the ambient noise $n(k)$ are assumed to be known. However, in echo cancellers, it is impractical to know $\mathbf{Q}(k)$ and $R(k)$. Therefore, it is difficult to apply the Kalman filter directly to acoustic echo cancellers.

3.2 Kalman filter and RLS algorithm

The RLS algorithm was derived to recursively solve the least-squares estimation problem. It was originally independent of the Kalman filter. However, the RLS algorithm can be derived from the Kalman filter (1)–(5) as follows [2].

First, $\mathbf{Q}(k)$ in (4) is assumed to be

$$\mathbf{Q}(k) = (\nu^{-1} - 1)\tilde{\mathbf{P}}_K(k), \tag{6}$$

where $0 < \nu \leq 1$. Substituting (6) into (4) yields

$$\mathbf{P}_K(k+1) = \nu^{-1}\tilde{\mathbf{P}}_K(k). \tag{7}$$

Assumption (6) means that the coefficient error covariance matrix $\tilde{\mathbf{P}}_K(k)$ increases by $\nu^{-1}(\geq 1)$ according to the impulse response variation.

Next, the ambient noise $n(k)$ is assumed to be stationary, having time-invariant power $R(k) \equiv R$. Introducing a matrix $\mathbf{P}(k)$, which is related to $\mathbf{P}_K(k)$ by

$$\mathbf{P}_K(k) = \nu^{-1}R\,\mathbf{P}(k), \tag{8}$$

and substituting (3) and (6) into (4), and (8) into (2) and (4), the RLS algorithm (9)–(12) can be derived.

$$\hat{\mathbf{h}}(k+1) = \hat{\mathbf{h}}(k) + \mathbf{k}(k)e(k) \tag{9}$$

$$\mathbf{k}(k) = \frac{\nu^{-1}\mathbf{P}(k)\mathbf{x}(k)}{1 + \nu^{-1}\mathbf{x}(k)^T\mathbf{P}(k)\mathbf{x}(k)} \tag{10}$$

$$\mathbf{P}(k+1) = \nu^{-1}\mathbf{P}(k) - \nu^{-1}\mathbf{k}(k)\mathbf{x}(k)^T\mathbf{P}(k) \tag{11}$$

$$e(k) = y(k) - \hat{\mathbf{h}}(k)^T\mathbf{x}(k) + n(k). \tag{12}$$

Thus, the RLS algorithm can be derived from the Kalman filter by assuming the covariance matrix $\mathbf{Q}(k)$ of the impulse response variation of (6) and stationary ambient noise $R(k) \equiv R$.

4. NEW ADAPTIVE ALGORITHM

4.1 Variation in a room impulse response

An adaptive algorithm with suitable special assumptions about the characteristics of the " unknown system " to be identified is expected to improve convergence.

Although the detailed waveform is complicated, the envelope of a room impulse response (our " unknown system ") attenuates exponentially, and more importantly, the expected variation of the room impulse response also attenuates by the same exponential ratio when a person moves or the environment changes [4]. This is expressed by the equation:

$$E[\Delta h_i(k)^2] = \alpha_0(k)\gamma^{i-1} \quad (i = 1, \cdots, L), \tag{13}$$

where γ $(0 < \gamma \leq 1)$ is the exponential attenuation ratio of the room impulse response power. The value of γ is common to all impulse responses in the same room. It can be derived from the reverberation time, which is determined by the acoustical conditions of the room, *i.e.*, size and absorption coefficient. Thus, we can estimate the exponential attenuation ratio γ from the room conditions, or determine it by measuring one impulse response.

4.2 Exponentially weighted stepsize RLS (ES-RLS) algorithm

The ES-RLS algorithm is derived from the Kalman filter by introducing several assumptions as follows. First, each element of the impulse response variation $\Delta\mathbf{h}(k)$ is assumed to be a statistically independent random variable. As a result, the covariance matrix $\mathbf{Q}(k)$ of the variation $\Delta\mathbf{h}(k)$ becomes a diagonal matrix. Then, for the diagonal component of the matrix $\mathbf{Q}(k)$, *i.e.*, $E[\Delta h_i(k)^2]$, we use (13) which reflects the variation characteristics of the room impulse response. The $\alpha_0(k)$, which represents the magnitude of the variation, is assumed to take time-invariant value α_0. Based on these assumptions, we set $\mathbf{Q}(k)$ as

$$\mathbf{Q}(k) \equiv \mathbf{A} = \begin{pmatrix} \alpha_1 & & & 0 \\ & \alpha_2 & & \\ & & \ddots & \\ 0 & & & \alpha_L \end{pmatrix} \tag{14}$$

where

$\alpha_i = \alpha_0 \gamma^{i-1} \quad (i = 1, \cdots, L)$,

γ : exponential attenuation ratio of room impulse responses ($0 < \gamma \le 1$).

Elements α_i are time-invariant and decrease exponentially from α_1 to α_L by the same ratio γ as the impulse response $\mathbf{h}(k)$.

Here we define the mean stepsize $\bar{\alpha}$ which represents the magnitude of the matrix $\mathbf{A}$ as

$$\bar{\alpha} = \frac{1}{L}\sum_{i=1}^{L}\alpha_i = \frac{\alpha_0}{L}\frac{1-\gamma^L}{1-\gamma}. \tag{15}$$

Then, assuming that the ambient noise $n(k)$ is stationary $[R(k) \equiv R]$, we introduce $\mathbf{P}_{ES}(k)$ by multiplying the a priori coefficient error covariance matrix $\mathbf{P}_K(k)$ of the Kalman filter by $1/R$, *i.e.*,

$$\mathbf{P}_K(k) = R\ \mathbf{P}_{ES}(k). \tag{16}$$

Substituting (16) into (2), and using $R(k) \equiv R$, and then substituting (3), (14), and (16) into (4), we get the following ES-RLS algorithm [7].

$$\hat{\mathbf{h}}(k+1) = \hat{\mathbf{h}}(k) + \mathbf{k}(k)e(k) \tag{17}$$

$$\mathbf{k}(k) = \frac{\mathbf{P}_{ES}(k)\mathbf{x}(k)}{1 + \mathbf{x}(k)^T\mathbf{P}_{ES}(k)\mathbf{x}(k)} \tag{18}$$

$$\mathbf{P}_{ES}(k+1) = \mathbf{P}_{ES}(k) - \mathbf{k}(k)\mathbf{x}(k)^T\mathbf{P}_{ES}(k) + \frac{\mathbf{A}}{R} \tag{19}$$

$$e(k) = y(k) - \hat{\mathbf{h}}(k)^T\mathbf{x}(k) + n(k) \tag{20}$$

where

$\mathbf{P}_{ES}(k) : L \times L$ matrix,

$\mathbf{A}$: stepsize matrix.

Elements [$\alpha_1, \alpha_2, ..., \alpha_L$] of the stepsize matrix $\mathbf{A}$ are not really "stepsizes" like in the conventional NLMS or projection algorithms. However, as described below, these elements function as if they were stepsizes, and from the relationship between the previously proposed ES algorithm [4] and ESP algorithm [5], we call matrix $\mathbf{A}$ a stepsize matrix.

On the other hand, the stepsize is known to be related to the forgetting factor ν of the RLS algorithm. In fact, according to (19), when the value $\mathbf{A}/R$ is large compared to $\mathbf{P}_{ES}(k)$, the proportion of $\mathbf{P}_{ES}(k)$ in $\mathbf{P}_{ES}(k+1)$ becomes small. In other words, old information is forgotten quickly. Thus, the mean stepsize $\bar{\alpha}$ shown in (15) has the same role as the forgetting factor ν of the RLS algorithm.

The covariance matrix $\mathbf{Q}(k)$ of the impulse response variation is added in (4); on the other hand, the time-invariant $\mathbf{A}/R$ is always added in (19). In other words, exponentially attenuating bias is always added in diagonal elements of the matrix $\mathbf{P}_{ES}(k)$. As a result, the gain vector $\mathbf{k}(k)$ attenuates exponentially in (18), so the filter coefficient vector $\hat{\mathbf{h}}(k)$ is adjusted by the exponentially attenuating adjustment vector in (17). Accordingly, this algorithm adjusts coefficients with large errors in large steps and coefficients with small errors in small steps.

5. COMPUTER SIMULATIONS

Computer simulation results on the $ERLE$ convergence of the ES-RLS algorithm are shown in Figs. 2 and 3. The exponentially attenuating impulse response was generated by exponentially windowed white noise in the computer. The number of taps was 64. The filter coefficients were initially set to zero. Ambient noise with a fixed SNR of 17 dB was added. The impulse response changed at time step $k = 1000$. $ERLE$ (echo return loss enhancement) is defined by

$$ERLE = 10\ \log_{10}\frac{p_y}{p_e} \quad \text{(dB)}, \tag{21}$$

where

$p_y = E[y(k)^2]$,

$p_e = E[\tilde{e}(k)^2] = E[(y(k) - \hat{y}(k))^2]$.

The signal powers p_y and p_e were estimated from 10 data samples for white noise and 256 for speech. Each curve is the average of 50 independent results.

In the proposed algorithm, $\mathbf{A}/R$ is added to matrix $\mathbf{P}_{ES}(k)$ at every time step k. This results in fast convergence because the expected variation of the room impulse response is reflected when the impulse response varies. However, this means that unnecessary variation (noise) is added even when the impulse response is not varying. As a result, the steady-state $ERLE$ stays at a low level due to this noise. Such an effect is expected to be significant when the magnitude of the stepsize matrix $\mathbf{A}$ (mean stepsize $\bar{\alpha}$) is large, *i.e.*, a larger $\bar{\alpha}$ results in a faster convergence but a smaller steady-state $ERLE$. On the other hand, a smaller $\bar{\alpha}$ results in slower convergence but a larger steady-state $ERLE$. Figure 2 demonstrates this effect, where parameter $\bar{\alpha}/R$ was varied. The received input was white noise. As stated above, the mean stepsize $\bar{\alpha}$ controls the trade-off between the convergence speed and the steady-state $ERLE$.

Next, Fig. 3 shows the $ERLE$ convergence in the ES-RLS algorithm and the conventional RLS algorithm. The received inputs were (a) white noise and (b) speech. The forgetting factor of the RLS algorithm was set to $\nu = 0.998$ to get a steady-state $ERLE$ of 30 dB. In the ES-RLS algorithm, the mean stepsize $\bar{\alpha}$ was set to get the same value of the steady-state $ERLE$ as the conventional RLS algorithm. With both a white noise input [Fig. 3 (a)] and a speech input [Fig. 3 (b)], the ES-RLS algorithms reaches an $ERLE$ of 20 dB twice as fast as the RLS algorithm.

The ES-RLS algorithm is regarded as an algorithm that replaces the covariance matrix $\mathbf{Q}(k)$ of the impulse response variation in the Kalman filter by the time-invariant exponentially weighted diagonal matrix $\mathbf{A}$. This replacement assumes that the room impulse response always varies by $\mathbf{A}$. On the other hand, computer simulations, where the impulse response changed only at time step $k = 1000$, are

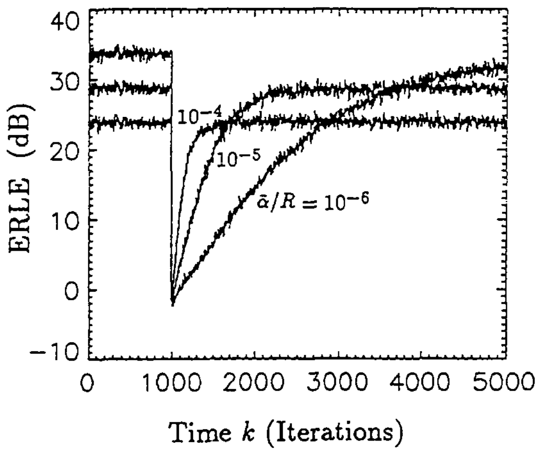

Fig. 2 Computer simulation results on *ERLE* convergence showing the effect of mean stepsize $\bar{\alpha}$. The mean stepsize $\bar{\alpha}$ controls the trade-off between the convergence speed and the steady-state *ERLE*.

quite different from the assumption of the impulse response variation in the ES-RLS algorithm. Nevertheless, the ES-RLS algorithm was shown to have double the convergence speed of the RLS algorithm.

6. CONCLUSIONS

Knowledge about a room impulse response has been incorporated into the conventional RLS algorithm, which is the fastest of all conventional adaptive algorithms. As a result, we have proposed a new adaptive algorithm, called the ES-RLS (exponentially weighted stepsize RLS) algorithm.

The expected variation of a room impulse response becomes progressively smaller along the series by the same exponential ratio as the impulse response energy decay. The ES-RLS algorithm is derived by incorporating these variation characteristics of the room impulse response into the updating equation of the coefficient error covariance matrix of the conventional RLS algorithm using Kalman filter theory, which gives physical meaning to the RLS algorithm. A diagonal matrix (stepsize matrix **A**) is added to the covariance matrix. The diagonal components of this stepsize matrix are time-invariant and are set proportional to the expected variation of the room impulse response.

The magnitude of the stepsize matrix (mean stepsize $\bar{\alpha}$) controls the trade-off between the convergence speed and the steady-state *ERLE*. Computer simulations showed that our new adaptive algorithm has double the convergence speed of the conventional RLS algorithm.

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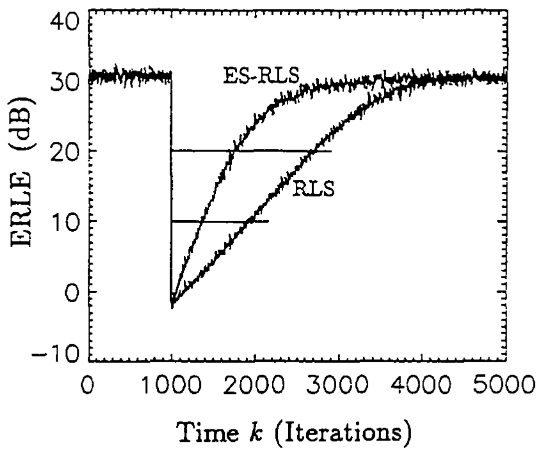
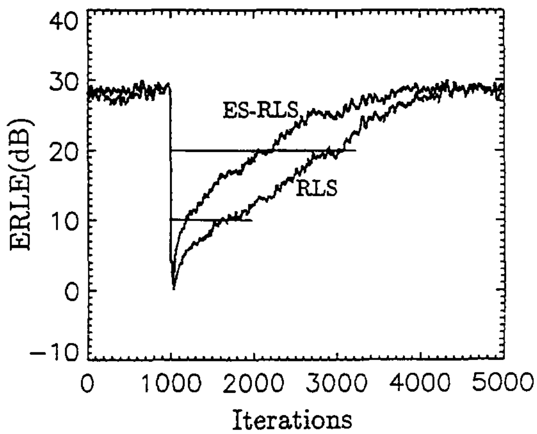


(a) Input signal: white noise



(b) Input signal: speech (male)

Fig. 3 Computer simulation results on *ERLE* convergence showing comparison with the conventional RLS algorithm. The proposed ES-RLS algorithm converges twice as fast as the RLS algorithm.

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