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Stereo echo cancellation algorithm using adaptive update on the basis of enhanced input-signal vector

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Abstract

Stereo echo cancellation requires a fast converging adaptive algorithm because the stereo input signals are highly cross correlated and the convergence rate of the misalignment is slow even after preprocessing for unique identification of stereo echo paths. To speed up the convergence, we propose enhancing the contribution of the decorrelated components in the preprocessed input-signal vector to adaptive updates. The adaptive filter coefficients are updated on the basis of either a single or multiple past enhanced input-signal vectors.

For a single-vector update, we show how this enhancement improves the convergence rate by analyzing the behavior of the filter coefficient error in the mean. For a two-past-vector update, simulation showed that the proposed enhancement leads to a faster decrease in misalignment than the corresponding conventional second-order affine projection algorithm while computational complexities are almost the same.
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1. Introduction

Advanced teleconferencing features full duplex communication and stereophonic sound transfer. These features lead to a greater sense of presence and more effective audio/video communication between participants, but they also make acoustic stereo echo cancellation indispensable.

The fundamental problem in acoustic stereo echo cancellation is the unique identification of the receiving room's impulse responses. Since the signals reproduced by the two loudspeakers in the receiving room are usually linearly related to each other [1–5], the adaptive filter for stereo echo cancellation faces an almost singular normal equation. This problem is of particular concern because lacking proper identification, echo cancellation becomes dependent on the impulse responses of the transmission room. This implies that one must track not only changes in the receiving room but also changes in the transmission room, which can be very rapid (e.g., when one person stops talking and another starts).

Injection of uncorrelated components into received signals has been proposed as a way of overcoming this problem [1–9]. Uncorrelated components should be injected at a level low enough so that they are not noticeable in the speech.

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However, this approach is only effective in combination with a fast converging adaptive algorithm because the adaptive algorithm must still solve the ill-conditioned normal equation even after the preprocessing.

The high computational expense of fast-converging time-domain adaptive algorithms is well known. The two-channel fast recursive least-squares (FRLS) algorithm is seven times as computationally intensive as the conventional two-channel normalized least-mean-square (NLMS) algorithm [10]. The high-order affine projection algorithm (APA) is also more computationally intensive than the conventional NLMS: in this case, it is proportional to the square of the projection order [11]. The fast converging sub-band and frequency-domain adaptive algorithms inevitably incur processing delay, although both are much less computationally complex than the NLMS algorithm [12,13].

In this paper, we propose updating the adaptive filter on the basis of an enhanced input-signal vector for obtaining a novel fast-converging adaptive algorithm. The basic idea of the algorithm is to generate an increment vector, i.e., a vector used to update the adaptive filter, with a smaller inter-channel cross-correlation than the preprocessed stereo signal itself [14,15].

The enhanced NLMS is obtained by replacing the gradient vector in the NLMS algorithm with an enhanced input-signal vector in which the uncorrelated component introduced for unique identification is enhanced by a simple operation. We extend the enhanced NLMS to the adaptive update on multiple bases to improve the convergence rate. This generalized version of the enhanced NLMS (GENLMS) is obtained from APA in the same manner. Simulation showed that the second-order GENLMS decreased the misalignment between the estimated and true echo paths more quickly while being computationally as efficient as APA with the same low order.

This paper is organized as follows. In Section 2, the NLMS and APA are reviewed as two conventional adaptive algorithms for stereo echo cancellation. The enhanced NLMS is presented in Section 3. The convergence properties of the enhanced NLMS are discussed in Section 4. In Section 5, the enhanced NLMS is extended to the adaptive update on multiple bases. Section 6 presents simulation results that demonstrate the effectiveness of enhancing the injected uncorrelated component in the increment vector of the filter coefficients.

2. Conventional adaptive algorithms and their problems

Fig. 1 schematically shows a typical system for stereo echo cancellation. In the transmission room, stereo signals $u_1(k)$ and $u_2(k)$ are picked up by the two microphones and transmitted. In the receiving room, low-level uncorrelated components $v_1(k)$ and $v_2(k)$ are injected into the received stereo signals to avoid the non-uniqueness problem:

$$\begin{aligned} x_1(k) &= u_1(k) + v_1(k), \\ x_2(k) &= u_2(k) + v_2(k). \end{aligned} \tag{1}$$

The signal $y(k)$ at the microphone is expressed by

$$y(k) = \mathbf{h}^{\mathrm{T}}\mathbf{x}(k), \tag{2}$$

where $\mathbf{h} = [\mathbf{h}_1^{\mathrm{T}}\mathbf{h}_2^{\mathrm{T}}]^{\mathrm{T}}$ is the concatenation of the vectors of the true impulse responses $\mathbf{h}_1$ and $\mathbf{h}_2$. The input-signal vector $\mathbf{x}(k)$ is defined as

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1(k) \\ \mathbf{u}_2(k) \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1(k) \\ \mathbf{v}_2(k) \end{bmatrix}, \tag{3}$$

where

$$\mathbf{u}_i(k) = [u_i(k) \ \cdots \ u_i(k-L+1)]^{\mathrm{T}}, \quad i = 1, 2,$$

$$\mathbf{v}_i(k) = [v_i(k) \ \cdots \ v_i(k-L+1)]^{\mathrm{T}}, \quad i = 1, 2.$$

Here, L is the length of the impulse-response vectors.

The error $e(k)$ is expressed by

$$e(k) = y(k) - \hat{\mathbf{h}}^{\mathrm{T}}(k)\mathbf{x}(k), \tag{4}$$

where $\hat{\mathbf{h}}(k) = [\hat{\mathbf{h}}_1^{\mathrm{T}}(k) \ \hat{\mathbf{h}}_2^{\mathrm{T}}(k)]^{\mathrm{T}}$ is the concatenation of the vectors of adaptive filter coefficients $\hat{\mathbf{h}}_1(k)$ and $\hat{\mathbf{h}}_2(k)$. Note that, for simplicity, we only consider the acoustic paths to one microphone in the receiving

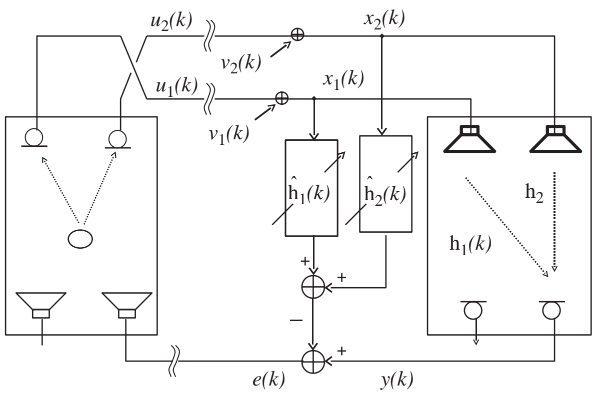

Fig. 1. Typical stereo echo cancellation system.

room; a similar analysis will apply to the other microphone.

In the stereo echo cancellation system shown in Fig. 1, the two sides of the original stereo signal from the transmission room will obviously have strong inter-channel cross-correlation because both originate from the same source, the single talker. The preprocessed stereo signal still has strong inter-channel cross-correlation since effects of preprocessing must be inaudible and the level of uncorrelated components is restricted.

In conventional two-channel NLMS, the vector of adaptive filter coefficients is updated in the following way:

$$\hat{\mathbf{h}}(k+1) = \hat{\mathbf{h}}(k) + \mu\Delta\hat{\mathbf{h}}(k),$$
$$\Delta\hat{\mathbf{h}}(k) = \frac{e(k)}{\mathbf{x}_1^{\mathrm{T}}(k)\mathbf{x}_1(k) + \mathbf{x}_2^{\mathrm{T}}(k)\mathbf{x}_2(k) + \delta} \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix}. \tag{5}$$

The directional component of the increment vector obviously contains no cross term involving both $\mathbf{x}_1(k)$ and $\mathbf{x}_2(k)$. Since the strong inter-channel cross-correlation of the preprocessed stereo signal remains in the increment vector as it is, the convergence of misalignment is slow [2].

In conventional two-channel APA, the increment vector consists of multiple input-signal vectors with strong inter-channel cross-correlation. As with NLMS, the increment vector's directional component contains no cross term involving $\mathbf{x}_1(k)$ and $\mathbf{x}_2(k)$:

$$\Delta\hat{\mathbf{h}}(k) = c_0(k)\begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix} + \cdots + c_{p-1}(k)\begin{bmatrix} \mathbf{x}_1(k-p+1) \\ \mathbf{x}_2(k-p+1) \end{bmatrix}. \tag{6}$$

The weights $c_0(k), \ldots, c_{p-1}(k)$ are obtained as

$$\begin{bmatrix} c_0(k) \\ \vdots \\ c_{p-1}(k) \end{bmatrix} = [\mathbf{X}^{\mathrm{T}}(k)\mathbf{X}(k)]^{-1} \times \begin{bmatrix} y(k) - \mathbf{x}_1^{\mathrm{T}}(k)\hat{\mathbf{h}}(k) \\ \vdots \\ y(k-p+1) - \mathbf{x}_1^{\mathrm{T}}(k-p+1)\hat{\mathbf{h}}(k) \end{bmatrix},$$

where

$$\mathbf{X}(k) = [\mathbf{x}(k), \ldots, \mathbf{x}(k-p+1)].$$

When the projection order p is low, the inter-sample correlation is largely removed, but the inter-channel cross-correlation partially remains in the increment vector. This inter-channel cross-correlation slows the convergence of misalignment.

3. New adaptive algorithm

We propose an enhanced form of the NLMS (enhanced NLMS) that overcomes the problem of slow convergence in this section.

The basic idea of this algorithm is to increase the contribution of injected components to the increment vector. The injected components $v_1(k)$ and $v_2(k)$ are designed to introduce uncorrelated components into the linearly correlated original input signals $u_1(k)$ and $u_2(k)$ [1]. This enhancement should reduce the inter-channel cross-correlation in the increment vector, and in turn improve the rate of convergence.

The enhanced NLMS is derived by replacing the increment vector for the NLMS algorithm:

$$\mu\Delta\hat{\mathbf{h}}(k) = \mu\mathbf{x}(k)\,\frac{e(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{x}(k) + \delta}, \tag{7}$$

where

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{u}_1(k) \\ \mathbf{u}_2(k) \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1(k) \\ \mathbf{v}_2(k) \end{bmatrix}, \tag{8}$$

with a new increment vector:

$$\mu\Delta\hat{\mathbf{h}}(k) = \mu\mathbf{z}(k)\,\frac{e(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k) + \delta}, \tag{9}$$

where

$$\mathbf{z}(k) = \begin{bmatrix} \mathbf{u}_1(k) \\ \mathbf{u}_2(k) \end{bmatrix} + \sigma\begin{bmatrix} \mathbf{v}_1(k) \\ \mathbf{v}_2(k) \end{bmatrix} \quad (\sigma > 1) \tag{10}$$

and δ is a small positive constant used to avoid division by 0.

An enhancement factor $\sigma(>1)$ is used to control the contribution of the uncorrelated components to the increment vector $\mu\Delta\hat{\mathbf{h}}(k)$, while keeping their contribution constant in the stereo loudspeaker signal $\mathbf{x}(k)$. The larger σ in the new increment vector increases the contribution of the uncorrelated components to adaptive updating. Note that the enhanced NLMS algorithm with $\sigma = 1$ is identical to the conventional NLMS algorithm.

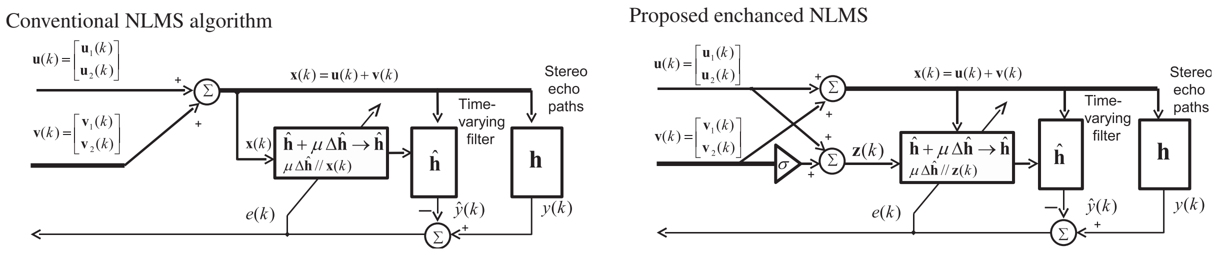

Fig. 2. Signal flow in the conventional NLMS algorithm and the proposed enhanced NLMS, where $\mu\Delta\hat{\mathbf{h}}(k)//\mathbf{x}(k)$ means that the direction of $\mu\Delta\hat{\mathbf{h}}(k)$ is the same as that of $\mathbf{x}(k)$. The increment vector $\mu\Delta\hat{\mathbf{h}}(k)$ for the enhanced NLMS is generated from enhanced input-signal vector $\mathbf{z}(k)$, where the enhancement factor σ controls the contribution of uncorrelated components $\mathbf{v}(k)$ introduced by preprocessing.

This algorithm can be derived by obtaining the weight $c_0(k)$ for

$$\Delta\hat{\mathbf{h}}(k) = \mathbf{z}(k)c_0(k) \tag{11}$$

that satisfies

$$y(k) = [\hat{\mathbf{h}}(k) + \Delta\hat{\mathbf{h}}(k)]^{\mathrm{T}}\mathbf{x}(k). \tag{12}$$

Hence, the denominator of (9) is given as $\mathbf{x}^{\mathrm{T}}(k)\,\mathbf{z}(k) + \delta$, whereas that of (7) is $\mathbf{x}^{\mathrm{T}}(k)\,\mathbf{x}(k) + \delta$.

Fig. 2 shows the signal flow of the conventional NLMS algorithm and the proposed enhanced NLMS. In both cases, the input to the echo paths is the same stereo signal. This means that the same preprocessed stereo sound is reproduced by the loudspeakers. In addition, the input to the time-varying filter $\hat{\mathbf{h}}(k)$ is the same stereo-input signal vector, from which echo replica $\hat{y}(k)$ is generated. On the other hand, the increment vector $\mu\Delta\hat{\mathbf{h}}(k)$ of the enhanced NLMS is generated not from input-signal vector $\mathbf{x}(k)$ but from enhanced input-signal vector $\mathbf{z}(k)$, in which the contribution of the uncorrelated component is enhanced by the enhancement factor σ.

In the conventional NLMS algorithm, we can also enhance the uncorrelated component in the increment vector (7) by using large $v_1(k)$ and $v_2(k)$. However, amplifying these components of the stereo loudspeaker signals degrades the reproduced speech.

4. Properties of enhanced NLMS

In this section, we validate our idea of enhancing the injected uncorrelated component in the increment vector. Firstly, we investigate the behavior of the filter coefficient error in the mean and show that the enhanced NLMS converges and that its convergence rate is improved. Secondly, we investigate the behavior of the norm of the coefficient error, thus deriving the range where the enhancement is effective.

4.1. Assumptions

In the following analysis, we use low-level independent random noise as the uncorrelated component injected into each channel for correct identification of the echo paths.

We make the following assumptions about the injected signals $v_1(k)$ and $v_2(k)$.

(A1) Neither signal is correlated with the original stereo signal.

(A2) The signal power levels are given by

$$E\{v_1^2(k)\} = E\{v_2^2(k)\} = w, \tag{13}$$

where $E\{\cdot\}$ denotes mathematical expectation. We also assume:

(A3) The input signals $u_1(k)$ and $u_2(k)$ are zero-mean and wide-sense stationary. The covariance matrix is independent of time k and can be diagonalized by using the orthogonal matrix $\mathbf{A}$ ($\mathbf{A}^{\mathrm{T}}\mathbf{A} = \mathbf{A}\mathbf{A}^{\mathrm{T}} = \mathbf{I}_{2L}$) as

$$\begin{aligned}\mathbf{R}_{\mathbf{u}} &= E\left\{\begin{bmatrix}\mathbf{u}_1(k)\\ \mathbf{u}_2(k)\end{bmatrix}[\mathbf{u}_1^{\mathrm{T}}(k)\ \ \mathbf{u}_2^{\mathrm{T}}(k)]\right\}\\ &= \mathbf{A}^{\mathrm{T}}\,\mathrm{diag}(q_{2L},\ldots,q_1)\mathbf{A}\\ &\quad q_{2L}\geqslant\cdots\geqslant q_1\geqslant 0,\end{aligned} \tag{14}$$

where $\mathbf{I}_{2L}$ denotes the $2L\times 2L$ identity matrix.

Throughout this section, we assume that $0<\mu\leqslant 1$, because step size μ is usually set smaller than or equal to 1.

4.2. Transition of mean coefficient error

Let us study the behavior of the mean coefficient error of the adaptive filter in the transformed space. Under the above assumptions, this behavior is determined by a similar transition formula to that for the least-mean square (LMS) algorithm. We can analyze its convergence in the same way as that of the LMS [16,17].

The coefficient error in the transformed space is defined as

$$\mathbf{s}(k) = [s_1(k) \quad \cdots \quad s_{2L}(k)]^{\mathrm{T}} = \mathbf{A}[\mathbf{h} - \hat{\mathbf{h}}(k)]. \tag{15}$$

This vector of the coefficient error is updated by the enhanced NLMS in this way:

$$\mathbf{s}(k+1) = \mathbf{s}(k) - \mu\mathbf{A}\,\frac{\mathbf{z}(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)}e(k), \tag{16}$$

where the regularization is omitted for simplicity.

Substituting

$$e(k) = \mathbf{x}^{\mathrm{T}}(k)\mathbf{h} - \mathbf{x}^{\mathrm{T}}(k)\hat{\mathbf{h}}(k) = \mathbf{x}^{\mathrm{T}}(k)\mathbf{A}^{\mathrm{T}}\mathbf{s}(k) \tag{17}$$

into the update Eq. (16), we get the following transition formula [16,17].

$$\mathbf{s}(k+1) = \mathbf{T}_S(k)\mathbf{s}(k), \tag{18}$$

where

$$\mathbf{T}_S(k) = \mathbf{I}_{2L} - \mu\mathbf{A}\,\frac{\mathbf{z}(k)\mathbf{x}^{\mathrm{T}}(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)}\mathbf{A}^{\mathrm{T}}. \tag{19}$$

For large filter order $L \gg 1$, the squared norm $\mathbf{x}^{\mathrm{T}}(k)\mathbf{x}(k)$ of the stationary input-signal vector $\mathbf{x}(k)$ becomes almost independent of time, and can be approximated by a constant [18]. Similarly, $\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)$ can be approximated by a constant $Q + \sigma W$, where

$$Q = \text{trace of } \operatorname{diag}(q_{2L}, \ldots, q_1),$$
$$W = \text{trace of } w\,\mathbf{I}_{2L}.$$

See Appendix for details.

For a sufficiently small step size, the variations of the coefficient error vector are much slower than those of the input signal, so $\mathbf{z}(k)\mathbf{x}^{\mathrm{T}}(k)$ can be replaced with its ensemble average ("direct averaging", [16,18]). The mean transition matrix $E\{\mathbf{T}_S(k)\}$ is the following diagonal matrix:

$$\begin{aligned} E\{\mathbf{T}_S(k)\} &= \mathbf{I}_{2L} - \frac{\mu}{Q+\sigma W}\,\mathbf{A}E\{\mathbf{z}(k)\mathbf{x}^{\mathrm{T}}(k)\}\mathbf{A}^{\mathrm{T}} \\ &= \mathbf{I}_{2L} - \frac{\mu}{Q+\sigma W} \\ &\quad \times \begin{bmatrix} q_{2L}+\sigma w & & 0 \\ & \ddots & \\ 0 & & q_1+\sigma w \end{bmatrix}, \end{aligned} \tag{20}$$

4.3. Convergence in the mean

We can analyze the behavior of the mean coefficient error $E\{\mathbf{s}(k)\}$ by analyzing the largest eigenvalue of $E\{\mathbf{T}_S(k)\}$, which determines the convergence rate of $E\{\mathbf{s}(k)\}$.

The largest eigenvalue of $E\{\mathbf{T}_S(k)\}$ is given by

$$\lambda_{\max}(E\{\mathbf{T}_S(k)\}) = 1 - \mu\frac{q_1 + \sigma w}{Q + \sigma W}, \tag{21}$$

where q_1 is the smallest eigenvalue of the covariance matrix $\mathbf{R_u}$ of the stereo input signal before preprocessing.

This largest eigenvalue $\lambda_{\max}(E\{\mathbf{T}_S(k)\})$ satisfies

$$0 < \lambda_{\max}(E\{\mathbf{T}_S(k)\}) < 1, \tag{22}$$

since $0 < \mu \leqslant 1$ and $0 < \frac{q_1+\sigma w}{Q+\sigma W} < 1$.

Therefore, the coefficient error of the adaptive filter converges in the mean.

For the correlated stereo input signal, the smallest eigenvalue q_1 of the covariance matrix $\mathbf{R_u}$ is almost 0 [3]. The following approximation holds for the largest eigenvalue of $E\{\mathbf{T}_S(k)\}$.

$$\begin{aligned} \lambda_{\max}(E\{\mathbf{T}_S(k)\}) &\cong 1 - \mu\frac{\sigma w}{Q+\sigma W} \\ &= 1 - \mu\frac{w}{Q/\sigma + W}. \end{aligned} \tag{23}$$

This largest eigenvalue is monotonically decreasing for $1 \leqslant \sigma$. Hence, a larger enhancement factor σ can reduce the largest eigenvalue $\lambda_{\max}(E\{\mathbf{T}_S(k)\})$ and improve the convergence rate of the mean coefficient error.

4.4. Transition of coefficient error norm

Next, we investigate the range of σ where the enhancement is effective. For that purpose, we analyze the behavior of the norm of the coefficient-error vector. This norm must be decreasing for the convergence of the coefficient error.

The coefficient error vector is defined as

$$\mathbf{m}(k+1) = \mathbf{h} - \hat{\mathbf{h}}(k+1) \tag{24}$$

and its evolution is given as

$$\mathbf{m}(k+1) = \mathbf{m}(k) - \mu\,\frac{\mathbf{z}(k)e(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)}, \tag{25}$$

where the regularization is omitted for simplicity.

The evolution of its norm $\|\mathbf{m}(k)\|^2$ is given as

$$\begin{aligned}\|\mathbf{m}(k+1)\|^2 &= \left(\mathbf{m}(k) - \mu\,\frac{\mathbf{z}(k)e(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)}\right)^{\mathrm{T}} \\ &\quad\times\left(\mathbf{m}(k) - \mu\,\frac{\mathbf{z}(k)e(k)}{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)}\right) \\ &= \mathbf{m}^{\mathrm{T}}(k)\mathbf{m}(k) - 2\mu\,\mathbf{m}^{\mathrm{T}}(k)\frac{\mathbf{z}(k)e(k)}{Q+\sigma W} \\ &\quad + \mu^2\,\frac{Q+\sigma^2 W}{(Q+\sigma W)^2}e^2(k),\end{aligned} \tag{26}$$

where we have used the relations

$$\mathbf{z}(k)^{\mathrm{T}}\mathbf{x}(k) \cong Q+\sigma W,$$

$$\mathbf{z}(k)^{\mathrm{T}}\mathbf{z}(k) \cong Q+\sigma^2 W.$$

For large filter order $L \gg 1$, the squared norm $\mathbf{x}^{\mathrm{T}}(k)\mathbf{x}(k)$ of the stationary input-signal vector $\mathbf{x}(k)$ can be approximated by a constant [18]. Similarly, $\mathbf{z}(k)^{\mathrm{T}}\mathbf{z}(k)$ and $\mathbf{z}(k)^{\mathrm{T}}\mathbf{x}(k)$ can be approximated by a constant.

By substituting $e(k) = \mathbf{x}^{\mathrm{T}}(k)\,\mathbf{m}(k)$ into the above equation, we obtain the difference between the norms of the coefficient error vector as

$$\begin{aligned}&\|\mathbf{m}(k+1)\|^2 - \|\mathbf{m}(k)\|^2 \\ &\quad= \mathbf{m}^{\mathrm{T}}(k)\Bigg(-\frac{2\mu}{Q+\sigma W}\mathbf{z}(k)\mathbf{x}^{\mathrm{T}}(k) \\ &\qquad+\mu^2\frac{Q+\sigma^2 W}{(Q+\sigma W)^2}\,\mathbf{x}(k)\mathbf{x}^{\mathrm{T}}(k)\Bigg)\mathbf{m}(k).\end{aligned} \tag{27}$$

By taking the mathematical expectation, we can further rewrite the above equation as

$$\begin{aligned}&E\{\|\mathbf{m}(k+1)\|^2\} - E\{\|\mathbf{m}(k)\|^2\} \\ &\quad= E\Bigg\{\mathbf{m}^{\mathrm{T}}(k)\Bigg(-\frac{2\mu}{Q+\sigma W}E\{\mathbf{z}(k)\mathbf{x}^{\mathrm{T}}(k)\} \\ &\qquad+\mu^2\frac{Q+\sigma^2 W}{(Q+\sigma W)^2}E\{\mathbf{x}(k)\mathbf{x}^{\mathrm{T}}(k)\}\Bigg)\mathbf{m}(k)\Bigg\} \\ &\quad= E\Bigg\{\mathbf{m}^{\mathrm{T}}(k)\mathbf{A}^{\mathrm{T}}\frac{1}{Q+\sigma W}(-2\mu(\mathbf{Q}+\sigma w\mathbf{I}) \\ &\qquad+\mu^2\frac{Q+\sigma^2 W}{Q+\sigma W}(\mathbf{Q}+w\mathbf{I}))\mathbf{A}\mathbf{m}(k)\Bigg\},\end{aligned} \tag{28}$$

where we resort to the independence assumptions. That is, we assume that coefficient error vector $\mathbf{m}(k)$ is statistically independent of $\mathbf{x}(k)$ and $\mathbf{z}(k)$ [16,17]. We have also used the relations

$$E\{\mathbf{z}(k)\mathbf{x}(k)^{\mathrm{T}}\} = \mathbf{A}^{\mathrm{T}}(\mathbf{Q}+\sigma w\mathbf{I})\mathbf{A},$$

$$E\{\mathbf{x}(k)\mathbf{x}(k)^{\mathrm{T}}\} = \mathbf{A}^{\mathrm{T}}(\mathbf{Q}+w\mathbf{I})\mathbf{A}. \tag{29}$$

4.5. Contractivity on coefficient error norm

For the coefficient error vector to converge, the norm of coefficient error vector $\|\mathbf{m}(k)\|^2$ must decrease on average for every iteration step.

The difference of the mean norm can be rewritten as the weighted inner product of $(\mathbf{A}\mathbf{m}(k))$ by

$$\begin{aligned}&E\{\|\mathbf{m}(k+1)\|^2\} - E\{\|\mathbf{m}(k)\|^2\} \\ &\quad= E\left\{(\mathbf{A}\mathbf{m}(k))^{\mathrm{T}}\frac{\mathbf{P}}{Q+\sigma W}(\mathbf{A}\mathbf{m}(k))\right\},\end{aligned} \tag{30}$$

where

$$\begin{aligned}\mathbf{P} = &-2\mu(\mathbf{Q}+\sigma w\mathbf{I}) \\ &+\mu^2\,\frac{Q+\sigma^2 W}{Q+\sigma W}(\mathbf{Q}+w\mathbf{I}).\end{aligned} \tag{31}$$

The matrix $\mathbf{P}$ is diagonal, and its diagonal elements are given by

$$P_{jj} = -2\mu(q_j+\sigma w) + \mu^2\,\frac{Q+\sigma^2 W}{Q+\sigma W}(q_j+w). \tag{32}$$

For $E\{\|\mathbf{m}(k)\|^2\}$ to decrease, we must find the condition where all the diagonal elements $P_{jj}(1 \leqslant j \leqslant 2L)$ must be negative. If there exists P_{jj} greater than 0, then $E\{\|\mathbf{m}(k)\|^2\}$ is increasing and the adaptive filtering becomes unstable. The balance of the first and second terms determines whether P_{jj} is negative. The first term of P_{jj} is always negative in $1 \leqslant \sigma$, since $0<\mu\leqslant 1$, $q_j \geqslant 0$, and $w>0$. The second term is positive and monotonically increasing for $1\leqslant\sigma$, since $0<\mu\leqslant 1$, $Q>0$, $W>0$, $q_j\geqslant 0$, $w>0$, and

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\left(\frac{Q+\sigma^2 W}{Q+\sigma W}\right) = \frac{W^2\sigma^2 + WQ(2\sigma-1)}{(Q+\sigma W)^2} > 0.$$

Fig. 3sketches P_{jj}/w as a function of the enhancement factor σ for $q_j = w, 10w, 30w, 100w$, where step size μ is set to 0.3 and W/Q is set to $-25\,\mathrm{dB}$. This graph indicates that there exists a range of σ where $P_{jj}(1\leqslant j\leqslant 2L)$ is negative and the upper bound of this range is small for larger q_j. Note that q_{2L}/w is not less than $Q/W = 25\,\mathrm{dB} \simeq 316.2$, since

$$\frac{q_1+\cdots+q_{2L}}{2Lw} = \frac{Q}{W} \quad \text{and} \quad q_{2L} \geqslant \cdots \geqslant q_1 \geqslant 0.$$

By solving $P_{jj}<0$ $(1\leqslant j\leqslant 2L)$ for σ, we could obtain the range of σ as complicated functions of the eigenvalue q_j of the covariance matrix $\mathbf{R}_{\mathbf{u}}$ of stereo input signal. Instead, we obtain a narrower

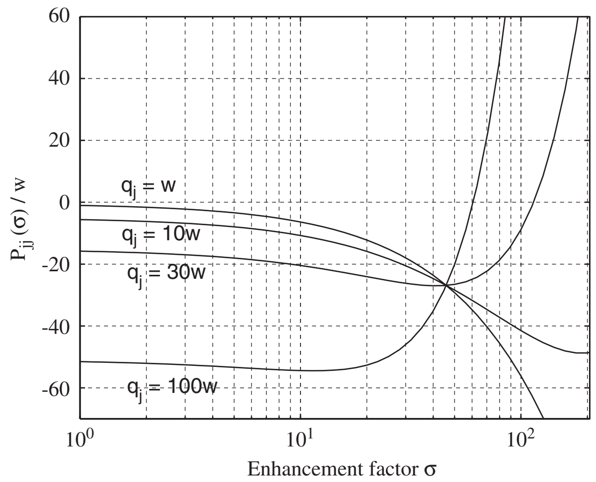

Fig. 3. P_{jj}/w as a function of the enhancement factor σ for various q_j.

(and easy-to-handle) range of σ by using the inequality

$$P_{jj} = -2\mu(q_j + \sigma w) + \mu^2 \frac{Q + \sigma^2 W}{Q + \sigma W}(q_j + w) < \mu(q_j + w)\left(\mu \frac{Q + \sigma^2 W}{Q + \sigma W} - 2\right) < 0. \tag{33}$$

From (33), we see that the mean norm of $E\{\|\mathbf{m}(k)\|^2\}$ decreases when σ satisfies

$$\mu \frac{Q + \sigma^2 W}{Q + \sigma W} - 2 \leqslant 0. \tag{34}$$

This condition is further rewritten in terms of σ as

$$1 \leqslant \sigma \leqslant \frac{1 + \sqrt{1 + \mu(2 - \mu)/W/Q}}{\mu}, \tag{35}$$

where W/Q is the ratio of the total power of injected signals to the total power of received signals.

5. Extension to the update on multiple bases

In this section, we extend the enhanced NLMS to the adaptive update on multiple bases as in the APA to improve the convergence rate. APA linearly combines multiple delayed input signal vectors to reduce the inter-sample correlation in the increment vector. Such a decorrelated increment vector improves the convergence rate for colored input signals [19,20].

We can easily apply this approach to the enhanced NLMS. We can derive a new adaptive algorithm by replacing the increment vector of APA, given by a linear combination of $\mathbf{x}(k), \ldots, \mathbf{x}(k-p+1)$, with a new increment vector given by a linear combination of $\mathbf{z}(k), \ldots, \mathbf{z}(k-p+1)$. Hereafter, we call this algorithm the generalized version of the enhanced NLMS (GENLMS).

The pth order GENLMS is represented by

$$\hat{\mathbf{h}}(k+1) = \hat{\mathbf{h}}(k) + \mu \Delta \hat{\mathbf{h}}(k), \tag{36}$$

where

$$\Delta \hat{\mathbf{h}}(k) = \mathbf{Z}(k)[\mathbf{X}^{\mathrm{T}}(k)\mathbf{Z}(k) + \delta \mathbf{I}]^{-1}\mathbf{e}(k),$$
$$\mathbf{X}(k) = [\mathbf{x}(k), \ldots, \mathbf{x}(k-p+1)],$$
$$\mathbf{Z}(k) = [\mathbf{z}(k), \ldots, \mathbf{z}(k-p+1)].$$

Here, $\mathbf{e}(k)$ is the error signal vector of dimension p, defined by

$$\mathbf{e}(k) = [y(k), \ldots, y(k-p+1)]^{\mathrm{T}} - \mathbf{X}^{\mathrm{T}}(k)\hat{\mathbf{h}}(k).$$

This algorithm is derived by obtaining weights $c_0(k), \ldots, c_{p-1}(k)$ for

$$\Delta \hat{\mathbf{h}}(k) = \mathbf{z}(k)c_0(k) + \cdots + \mathbf{z}(k-p+1)c_{p-1}(k) = \mathbf{Z}(k) \begin{bmatrix} c_0(k) \\ \vdots \\ c_{p-1}(k) \end{bmatrix} \tag{37}$$

that satisfy the last p input–output relationships

$$y(k) = \mathbf{x}^{\mathrm{T}}(k)[\hat{\mathbf{h}}(k) + \Delta \hat{\mathbf{h}}(k)],$$
$$\ldots,$$
$$y(k-p+1) = \mathbf{x}^{\mathrm{T}}(k-p+1)[\hat{\mathbf{h}}(k) + \Delta \hat{\mathbf{h}}(k)].$$

These p input–output relationships are equivalent to the following equation:

$$\mathbf{e}(k) = \mathbf{X}^{\mathrm{T}}(k)\Delta \hat{\mathbf{h}}(k) = \mathbf{X}^{\mathrm{T}}(k)\mathbf{Z}(k) \begin{bmatrix} c_0(k) \\ \vdots \\ c_{p-1}(k) \end{bmatrix}. \tag{38}$$

Hence, the weights $c_0(k), \ldots, c_{p-1}(k)$ are obtained as

$$\begin{bmatrix} c_0(k) \\ \vdots \\ c_{p-1}(k) \end{bmatrix} = [\mathbf{X}^{\mathrm{T}}(k)\mathbf{Z}(k)]^{-1}\mathbf{e}(k) \tag{39}$$

or, with a regularization constant δ, as

$$\begin{bmatrix} c_0(k) \\ \vdots \\ c_{p-1}(k) \end{bmatrix} = [\mathbf{X}^{\mathrm{T}}(k)\mathbf{Z}(k) + \delta\mathbf{I}]^{-1}\mathbf{e}(k). \tag{40}$$

Substituting (40) into (37) yields the increment. As shown in (37), this vector is a linear combination of vectors $\mathbf{z}(k), \ldots, \mathbf{z}(k-p+1)$, each of which has low inter-channel cross-correlation.

The computational complexity of GENLMS is almost the same as that of the conventional APA because we can easily generate vector $\mathbf{z}(k)$ from the signal samples $u_i(k) + \sigma v_i(k)$, $i = 1, 2$. There is no need for vector calculation to obtain vector $\mathbf{z}(k) = \mathbf{u}(k) + \sigma\mathbf{v}(k)$ for each sample.

6. Simulation

In this section, we report on our confirmation of the validity of the proposed idea through computer simulation. We investigated the behavior of the enhanced NLMS and second-order GENLMS.

We evaluated the performance of the proposed adaptive algorithm in terms of the misalignment as defined by

$$\frac{\|\mathbf{h} - \hat{\mathbf{h}}(k)\|^2}{\|\mathbf{h}\|^2}, \tag{41}$$

where $\mathbf{h}$ is the vector of truncated impulse responses of the receiving room. It is possible to have good echo cancellation even when the degree of misalignment is large. In such a case, however, the cancellation will become less effective if the impulse response in the transmission room changes.

6.1. Performance of the enhanced NLMS

Firstly, we investigated the convergence rate of the enhanced NLMS. We used samples of the 16 different colored noise sequences specified in the P50 recommendations (which are representative of the speech signal) [21] as the sound source in the transmission room. The received signals were correlated stereo pseudo speech signal followed by uncorrelated stereo white noise signal. The stereo pseudo speech signal was obtained by convolving the above sound source signals and transmission paths measured in a conference room with a reverberation time of 200 ms. The true echo paths were also measured in a conference room and then truncated to 2048 taps. The sampling frequency was 16 kHz.

We used independent random noises with equal power as the injected signal for preprocessing as in the analysis in Section 4. Their total power was set to –25 dB of the total power of the received signals ($W/Q = -25$ dB). The adaptive filter had 1536 taps on each channel. Step size μ was set to 0.3.

Fig. 4 shows the behavior of average misalignment for the enhanced NLMS with enhancement factor $\sigma = 1, 2, 4, 10$ (note that the enhanced NLMS with $\sigma = 1$ is identical to the conventional NLMS algorithm). We can see that a larger enhancement factor improved the convergence rate of misalignment in the enhanced NLMS. The dotted line shows the case of the NLMS algorithm without preprocessing, where the misalignment decreased rapidly in the first second, but approached saturation from about –3.5 dB. With preprocessing, the misalignment continued to decrease slowly, reaching around –4 dB at $t = 20$ s. This was improved to –10.5 dB by

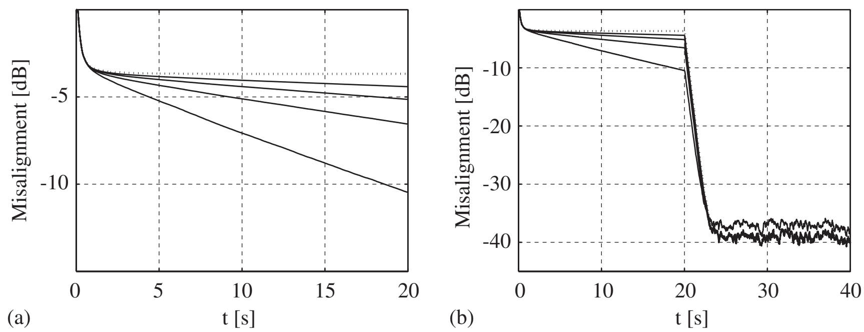

Fig. 4. Behavior of average misalignment for the conventional NLMS algorithm without preprocessing (dotted line) and the enhanced NLMS algorithm with preprocessing and various values of enhancement factor σ (the enhanced NLMS algorithm with $\sigma = 1$ is identical to the conventional NLMS algorithm). The inputs were (A) correlated stereo pseudo speech signals, (B) correlated stereo pseudo speech signals (0–20 s) and uncorrelated stereo white noise signal (20–40 s).

increasing σ to 10 in the enhanced NLMS. In each case, the final misalignment was around -40 dB at $t = 30$ s.

Next, we experimentally checked the validity of the convergence properties given in Section 4. For that purpose, we investigated the convergence behavior of the enhanced NLMS as a function of the enhancement factor σ when the step size μ was set to 0.1, 0.3, and 0.5. We used one received stereo signal in the previous simulation.

Fig. 5 shows the misalignment after 20-s adaptation for various values of σ. Each curve has a point where the misalignment was at its minimum. As σ increased from 1 to this point, the misalignment improved gradually. When σ exceeded this point, the misalignment deteriorated suddenly. The triangles indicate the theoretical upper bounds of the range of σ given by (35). We can see that the adaptive estimation did not deteriorate within this range. This result indicates that P_{jj} given by (32) increases rapidly and exceeds 0 when σ exceeds the upper bound of (35).

6.2. Performance of the enhanced NLMS for speech

Next, we investigate the convergence rate of the enhanced NLMS for speech and nonlinear preprocessing. The received signals were obtained by convolving a speech and transmission paths measured in a conference room with a reverberation time of 200 ms. The true echo paths were also measured in a conference room and then truncated to 2048 taps. The sampling frequency was 16 kHz.

We used half-wave rectifiers

$$v_1(k) = \alpha\,\frac{u(k) + |u(k)|}{2},$$
$$v_2(k) = \alpha\,\frac{u(k) - |u(k)|}{2} \tag{42}$$

with $\alpha = 0.3$ as nonlinear functions for preprocessing. For this value, it was reported that there was no audible degradation of the original signal [22]. A white-noise signal with 40-dB SNR was added as ambient noise to the signal arriving at the microphone, which then produced output signal $y(k)$. The adaptive filter had 1536 taps on each channel. Step size μ was set to 0.3.

Fig. 6 shows the behavior of misalignment for the NLMS and enhanced NLMS with enhancement factor $\sigma = 10$. We can see that the enhanced NLMS improved the convergence rate of misalignment. The dotted line shows the case of the NLMS algorithm without nonlinear preprocessing, where the misalignment decreased rapidly in the first second, but approached saturation from about –3.5 dB. With nonlinear preprocessing, the misalignment continued to decrease slowly, reaching around –6 dB at $t = 20$ s. This was improved to –9 dB by increasing σ to 10 in the enhanced NLMS.

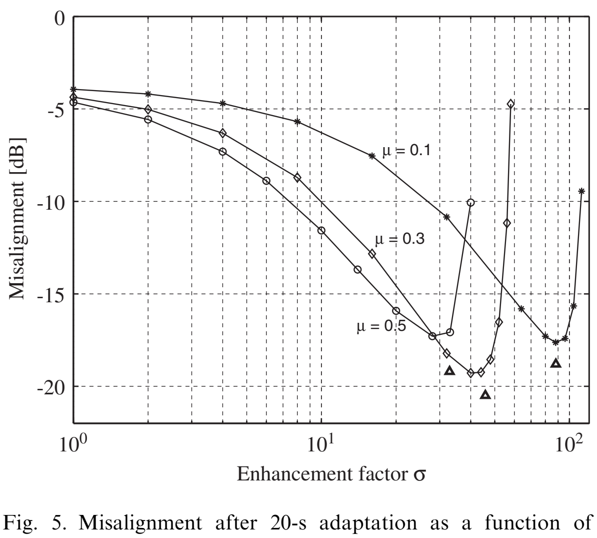

Fig. 5. Misalignment after 20-s adaptation as a function of enhancement factor σ. The triangles show the upper bounds of the range of σ for corresponding step sizes.

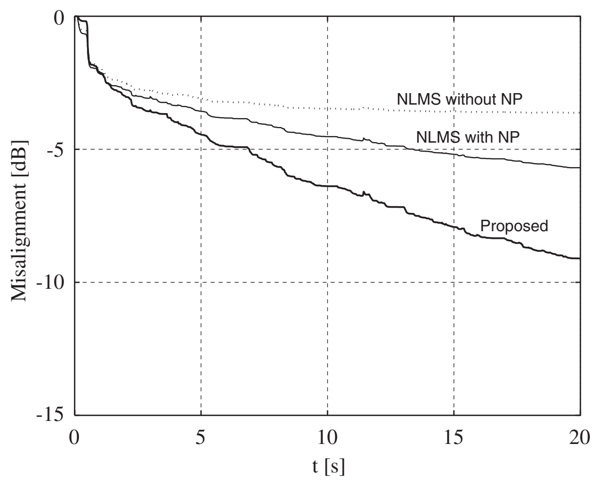

Fig. 6. Behavior of misalignment for the conventional NLMS algorithm with and without nonlinear preprocessing (without NP: dotted line) and the enhanced NLMS algorithm with enhancement factor $\sigma = 10$. The input was a correlated stereo speech signal. The nonlinearity gain α was 0.3.

6.3. Performance of the second-order GENLMS for speech

Next, we investigate the convergence rate of GENLMS for speech and nonlinear preprocessing. We used the same simulation settings as in the previous simulation.

Fig. 7 shows the behavior of misalignment for the conventional second-order APA and the second-order GENLMS ($p = 2$) with the enhancement factor $\sigma = 10$. We can see that the second-order GENLMS improved the convergence rate of misalignment. The dotted line shows the case of the conventional second-order APA without nonlinear preprocessing, where the misalignment again saturated at about –3.5 dB. With nonlinear preprocessing, the misalignment decreased slowly to –8.5 dB at $t = 20$ s. This was improved to –15.2 dB by increasing σ to 10 for the second-order GENLMS.

Note that the second-order GENLMS can be exactly implemented with the same computational complexity as that of NLMS by using the technique of the fast APA [23,24].

7. Conclusion

We have proposed updating the adaptive filter for stereo echo cancellation on the basis of an enhanced input-signal vector for obtaining a novel fast-converging adaptive algorithm. The increment vector used to update the adaptive filter enhances the uncorrelated component injected for unique identification of the echo paths. We showed how this enhancement is effective by analyzing the statistical behavior of the enhanced NLMS. Next, we extended the enhanced NLMS to the generalized version of the enhanced NLMS (GENLMS), where the adaptive filter is updated on multiple bases as in the APA to improve the convergence rate.

Simulation demonstrated that the two-channel second-order GENLMS decreased the misalignment of the adaptive filter more quickly than the conventional two-channel NLMS algorithm or the second-order APA, while having the same computational complexity as the two-channel second-order APA.

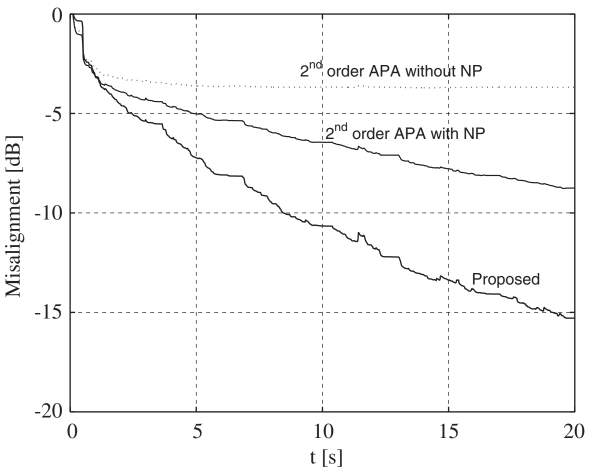

Fig. 7. Behavior of misalignment for the conventional second-order affine projection algorithm with and without nonlinear preprocessing (without NP: dotted line) and the second-order generalized version of the enhanced NLMS (GENLMS) with enhancement factor $\sigma = 10$. The input was a correlated stereo speech signal. The nonlinearity gain α was 0.3.

Appendix

In this appendix, we obtain the statistical properties of the signal vectors that appear in Section 4. Based on Assumptions (A1), (A2) and (A3), we obtain

$$E\left\{\begin{bmatrix}\mathbf{u}_1(k)\\ \mathbf{u}_2(k)\end{bmatrix}[\mathbf{v}_1^{\mathrm{T}}(k)\ \ \mathbf{v}_2^{\mathrm{T}}(k)]\right\} = \begin{bmatrix}\mathbf{0}_L & \mathbf{0}_L\\ \mathbf{0}_L & \mathbf{0}_L\end{bmatrix} \tag{43}$$

and

$$E\left\{\begin{bmatrix}\mathbf{v}_1(k)\\ \mathbf{v}_2(k)\end{bmatrix}[\mathbf{v}_1^{\mathrm{T}}(k)\ \ \mathbf{v}_2^{\mathrm{T}}(k)]\right\} = \begin{bmatrix}w\mathbf{I}_L & \mathbf{0}_L\\ \mathbf{0}_L & w\mathbf{I}_L\end{bmatrix}, \tag{44}$$

where $\mathbf{I}_L$ is the $L \times L$ identity matrix, and $\mathbf{0}_L$ is the $L \times L$ zero matrix.

In addition, we obtain

$$E\left\{[\mathbf{v}_1^{\mathrm{T}}(k)\ \ \mathbf{v}_2^{\mathrm{T}}(k)]\begin{bmatrix}\mathbf{v}_1(k)\\ \mathbf{v}_2(k)\end{bmatrix}\right\} = W \tag{45}$$

and

$$E\left\{[\mathbf{u}_1^{\mathrm{T}}(k)\ \ \mathbf{u}_2^{\mathrm{T}}(k)]\begin{bmatrix}\mathbf{v}_1(k)\\ \mathbf{v}_2(k)\end{bmatrix}\right\} = 0, \tag{46}$$

where

$W = \text{trace of } w\mathbf{I}_{2L}$.

From the assumptions about the stereo input signal that lead to (14), we obtain

$$E\left\{[\mathbf{u}_1^{\mathrm{T}}(k)\ \ \mathbf{u}_2^{\mathrm{T}}(k)]\begin{bmatrix}\mathbf{u}_1(k)\\ \mathbf{u}_2(k)\end{bmatrix}\right\} = Q, \tag{47}$$

where

$Q = \text{trace of } \mathrm{diag}(q_{2L}, \ldots, q_1)$.

Consequently, for

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{u}_1(k) \\ \mathbf{u}_2(k) \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1(k) \\ \mathbf{v}_2(k) \end{bmatrix} \quad \text{and} \quad \mathbf{z}(k) = \begin{bmatrix} \mathbf{u}_1(k) \\ \mathbf{u}_2(k) \end{bmatrix} + \sigma \begin{bmatrix} \mathbf{v}_1(k) \\ \mathbf{v}_2(k) \end{bmatrix},$$

we obtain

$$\mathbf{A}E\{\mathbf{x}(k)\mathbf{x}^{\mathrm{T}}(k)\}\mathbf{A}^{\mathrm{T}} = \mathrm{diag}(q_{2L}, \ldots, q_1) + w\mathbf{I}_{2L}, \tag{48}$$

$$\mathbf{A}E\{\mathbf{z}(k)\mathbf{z}^{\mathrm{T}}(k)\}\mathbf{A}^{\mathrm{T}} = \mathrm{diag}(q_{2L}, \ldots, q_1) + \sigma^2 w\mathbf{I}_{2L}, \tag{49}$$

$$\mathbf{A}E\{\mathbf{z}(k)\mathbf{x}^{\mathrm{T}}(k)\}\mathbf{A}^{\mathrm{T}} = \mathrm{diag}(q_{2L}, \ldots, q_1) + \sigma w\mathbf{I}_{2L}. \tag{50}$$

In addition,

$$E\{\mathbf{x}^{\mathrm{T}}(k)\mathbf{z}(k)\} = Q + \sigma W. \tag{51}$$

References

[1] M.M. Sondhi, D.R. Morgan, J.L. Hall, Stereophonic acoustic echo cancellation—an overview of the fundamental problem, IEEE Signal Process. Lett. 2 (8) (1995) 148–151.

[2] S. Shimauchi, S. Makino, Stereo projection echo canceller with true echo path estimation, in: Proceedings of ICASSP95, 1995, pp. 3059–3062.

[3] J. Benesty, D.R. Morgan, M.M. Sondhi, A better understanding and an improved solution to the problems of stereophonic acoustic echo cancellation, IEEE Trans. Speech Audio Process. 6 (2) (March 1998) 156–165.

[4] S. Makino, S. Shimauchi, Stereophonic acoustic echo cancellation—an overview and recent solutions, in: Proceedings of IWAENC99, 1999, pp. 941–944.

[5] E. Haensler, G. Schmidt, Acoustic Echo and Noise Control—A Practical Approach, Wiley, New York, 2004.

[6] Y. Joncour, A. Sugiyama, A stereo echo canceller with preprocessing for correct echo-path identification, in: Proceedings of ICASSP98, 1998, pp. 3677–3680.

[7] A. Gilloire, V. Turbin, Using auditory properties to improve the behavior of stereophonic acoustic echo cancellers, in: Proceedings of ICASSP98, 1998, pp. 3681–3683.

[8] M. Ali, Stereophonic acoustic echo cancellation system using time-varying all-pass filtering for signal decorrelation, in: Proceedings of ICASSP98, 1998, pp. 3689–3692.

[9] S. Shimauchi, S. Makino, Y. Haneda, A. Nakagawa, S. Sakauchi, A stereo echo canceller implemented using a stereo shaker and a duo-filter control system, in: Proceedings of ICASSP99, 1999, pp. 857–860.

[10] J. Benesty, F. Amand, A. Gilloire, Y. Grenier, Adaptive filtering algorithms for stereophonic acoustic echo cancellation, in: Proceedings of ICASSP95, 1995, pp. 3099–3102.

[11] F. Amand, J. Benesty, A. Gilloire, Y. Grenier, A fast two-channel projection algorithm for stereophonic acoustic echo cancellation, in: Proceedings of ICASSP96, 1996, pp. 949–952.

[12] S. Makino, K. Strauss, S. Shimauchi, Y. Haneda, A. Nakagawa, Subband stereo echo canceller using the projection algorithm with fast convergence to the true echo path, in: Proceedings of ICASSP97, 1997, pp. 299–302.

[13] J. Benesty, D.R. Morgan, Frequency domain adaptive filtering revisited, generalization to the multi-channel case, and application to acoustic echo cancellation, in: Proceedings of ICASSP2000, 2000, pp. 789–792.

[14] S. Emura, Y. Haneda, S. Makino, Adaptive algorithm enhancing decorrelated additive signals for stereo echo cancellation, in: Proceedings of IWAENC2001, 2001, pp. 67–70.

[15] A.W.H. Khong, P.A. Naylor, Selective-tap adaptive algorithms in solution of the nonuniqueness problem for stereophonic acoustic echo cancellation, IEEE Signal Process. Lett. 12 (4) (2005) 269–272.

[16] S. Haykin, Adaptive Filter Theory, third ed., Prentice-Hall, New Jersey, 1995.

[17] B. Widrow, S.D. Stearns, Adaptive Signal Processing, second ed., Prentice-Hall, New Jersey, 1985.

[18] H.J. Butterweck, Traveling-Wave Model of Long LMS Filters, Least-Mean-Square Algorithm, Wiley, New York, 2003, pp. 35–78.

[19] K. Ozeki, T. Umeda, An adaptive filtering algorithm using an orthogonal projection to an affine subspace and its properties, Electron. Commun. Jpn. 67-A (5) (1984) 19–27.

[20] S.G. Sankaran, A.A. Beex, Convergence behavior of affine projection algorithms, IEEE Trans. Signal Process. 48 (4) (2000) 1086–1096.

[21] R.G. 167, Performance of acoustic echo control devices, Technical Report, CCITT, Blue Book, ITU-TSS, 1992.

[22] P. Eneroth, T. Gänsler, S. Gay, J. Benesty, Studies of a wideband stereophonic acoustic echo canceller, in: Proceedings of IWAENC99, 1999, pp. 207–210.

[23] M. Tanaka, Y. Kaneda, S. Makino, J. Kojima, Fast projection algorithm and its step size control, in: Proceedings of ICASSP95, 1995, pp. 945–948.

[24] M. Tanaka, Y. Kaneda, S. Makino, J. Kojima, A fast projection algorithm for adaptive filtering, IEICE Trans. Fundamentals E78-A (10) (1995) 1355–1361.